Introduction to Radio Astronomy (WBAS14001)

Midterm examination – Tuesday 19 December 2017, 13:00 – 14:45

This examination contains 3 questions worth 20 points each. All questions should be answered. For full credit, all working and calculations should be shown, and full explanations given where required. Numerical answers should be in S.I. units, unless otherwise stated. Below is a list of constants and identities that may be used.

1. (a) Given that the infinitesimal power dP flowing through some infinitesimal surface-area $d\sigma$, over some infinitesimal bandwidth $d\nu$, within some infinitesimal solid angle $d\Omega$ from an incoming ray at angle θ of brightness I_ν, is given by

$$dP = I_\nu \cos\theta d\sigma d\Omega d\nu, \tag{1}$$

define in words what is meant by the flux-density and spectral intensity in terms of the power emitted, and derive equations for both [4 points].

(b) The observable radio window spans 5 decades in frequency due to the relative transparency of the atmosphere. Describe 4 factors that contribute to the varying transparency of the atmosphere at short radio wavelengths (1 to 100 GHz) and state their dependence on frequency [8 points].

(c) Draw the approximate *total atmospheric transparency* between 1 to 100 GHz, clearly labelling the two ranges in frequency where the atmospheric transparency to radio waves decreases most strongly [2 points].

(d) An observation of the continuum emission from the supernovae remnant Cassiopeia A is carried out with the Effelsberg radio telescope at a zenith angle of 48 degrees during sky conditions where the zenith optical depth is $\tau = 0.23$. Calculate the fraction of the emission from Cassiopeia A that is absorbed by the atmosphere during this observation [2 points].

(e) For the same observation described in part (d), what is the opacity of the atmosphere under the assumption that it is a constant slab of thickness 10 km [2 points].

(f) Give one example, under what circumstances you could increase the flux density of the emission received from Cassiopeia A, and explain your answer [2 points].

2. (a) Define in words what is meant by the *gain* of a dipole antenna [2 points].

(b) Give one example each for when it is useful have a high and a low gain dipole antenna [2 points].

(c) A dipole antenna has a directivity of 0.25 sr. Calculate the maximum gain of this antenna (in both linear and decibel units) [4 points].

(d) An observation of the radio galaxy Virgo A is carried out with the same dipole antenna described in part (c) at a frequency of 150 MHz. Calculate the maximum effective area of this dipole antenna [3 points].

(e) Define (in words) what is meant by *antenna temperature* (T_A) [2 points].

(f) Show that for an unpolarised source,

$$T_A = \frac{A_e S}{2k} \tag{2}$$

where A_e is the effective antenna area, and the other symbols have their usual meaning [3 points].

(g) Calculate the expected antenna temperature for an observation of Virgo A, using the dipole antenna described in part (c), given that at 150 MHz the flux-density of the target is 1200 Jy [2 points].

(h) Actually, the antenna temperature is found to be larger than this by a factor of 5. Give one possible explanation as to why this could be the case and explain your answer [2 points].

3. (a) Given that, for a one-dimensional filled aperture of length x, the electric field pattern in the far-field is the Fourier transform of the electric field illuminating (current grading) the aperture, such that,

$$f(l) = \int_{-\infty}^{+\infty} g(u)\, e^{-i2\pi l u}\, du, \tag{3}$$

where $u = x/\lambda$ and $l = \sin\theta$, show that for a one-dimensional aperture of diameter $D \gg \lambda$ and with a constant current grading, the normalised power pattern is given by,

$$P_n = \mathrm{sinc}^2\left(\frac{\theta D}{\lambda}\right) \qquad \text{[10 points].} \tag{4}$$

(b) Draw a schematic diagram of uniform, inverse Gaussian, and Gaussian current gradings, and their corresponding far-field power patterns, clearly showing their relative main beam widths and side-lobe structure [6 points].

(c) Calculate the full width at half power beam width for the 100 m diameter Effelsberg telescope at 5 GHz in arc-minutes, for a constant and Gaussian illumination [4 points].

Useful constants

Boltzmann constant $k = 1.3806488 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
Speed of light in a vacuum $c = 299\,792\,458$ m s^{-1}
1 Jansky (Jy) $\equiv 1 \times 10^{-26}$ W m^{-2} Hz^{-1}

Useful identities

$e^{i\phi} = \cos\phi + i\sin\phi$
The similarity theorem: if $f(l)$ is the Fourier transform of $g(u)$, then $(1/|a|)f(l/a)$ is the Fourier transform of $g(au)$, where a is a constant and not 0.